Unit I Algebraic

(Splitting field)

Defn An extension field K of a field F is said to be splitting field of a polynomial $f(x) \in F[x]$, if $f(x)$ considered over K is expressible as

$$f(x) = a(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots(x-\alpha_n),$$

where a is a non zero member of F and $\alpha_1, \alpha_2, \ldots, \alpha_n$ are in K such that

$$K = F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n).$$

x A finite extension K of a field F is said to be a splitting field over F if a polynomial $f(x)$ of degree $n \geq 1$ with coefficients in F [illegible]

(b) K is generated over F by the roots of $f(x)$ as

$$K = F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n).$$

Splitting field of a polynomial is also sometimes known as <u>Decomposition field</u> or <u>root field</u>.

Thus, given a polynomial of degree n over F there is a decomposition field of this polynomial which is a finite extension of F of degree at most $n!$ over F.

A decomposition field may also be defined as

"K is a decomposition field of $f(x)$ over F if K is a minimal extension of F in which $f(x)$ has n roots where n = degree $f(x)$."